

A Circle Proof

A point on a circle connected to the endpoints of a diameter forms a right triangle

Circle Properties

- ◆ tangent is perpendicular to radius at point of tangency.
- ◆ A point on a circle is connected to endpoints of a diameter, forming a right triangle.
- ◆ The central angle of an arc is double the inscribed angle of the same arc.

Proof: Construction of a right triangle.

Given:

Circle O with a diameter AB and point P .

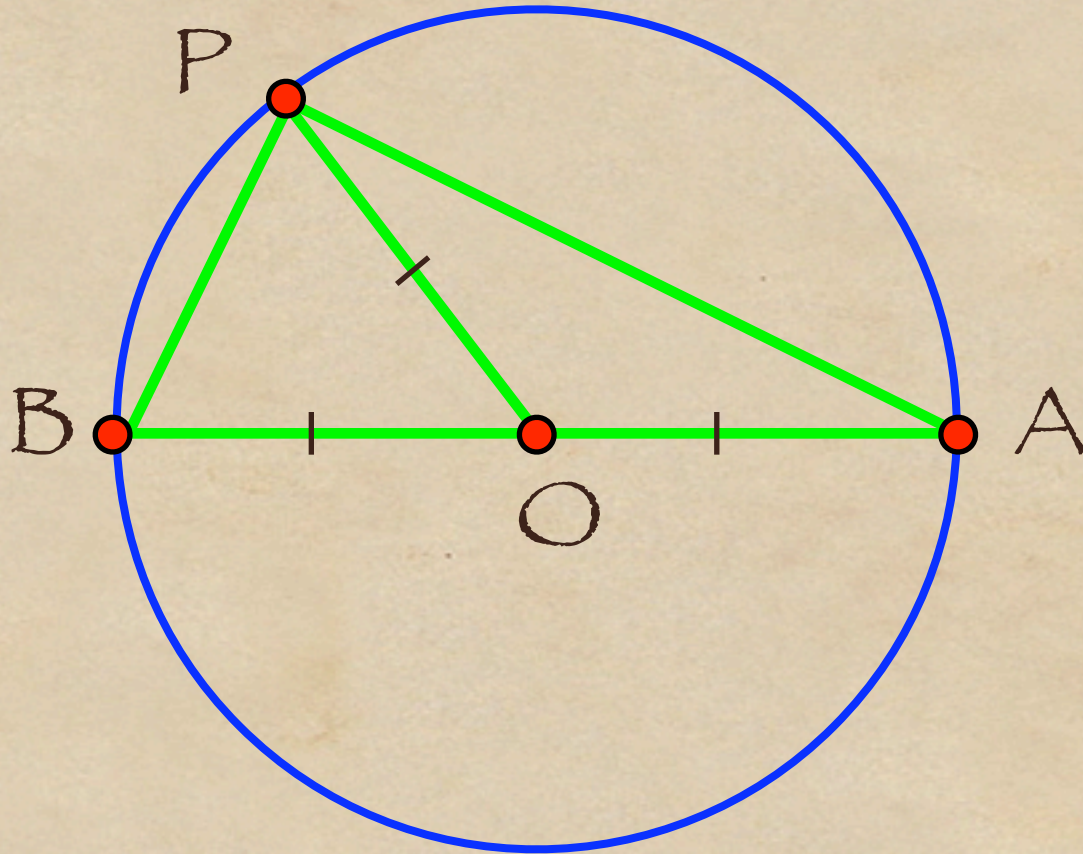
To Prove:

$\angle APB$ is a right angle.

There is an obvious line to be drawn, so we add OP .

OA , OP , and OB are radii of the circle and so are equal.

Triangles AOP and BOP are isosceles.



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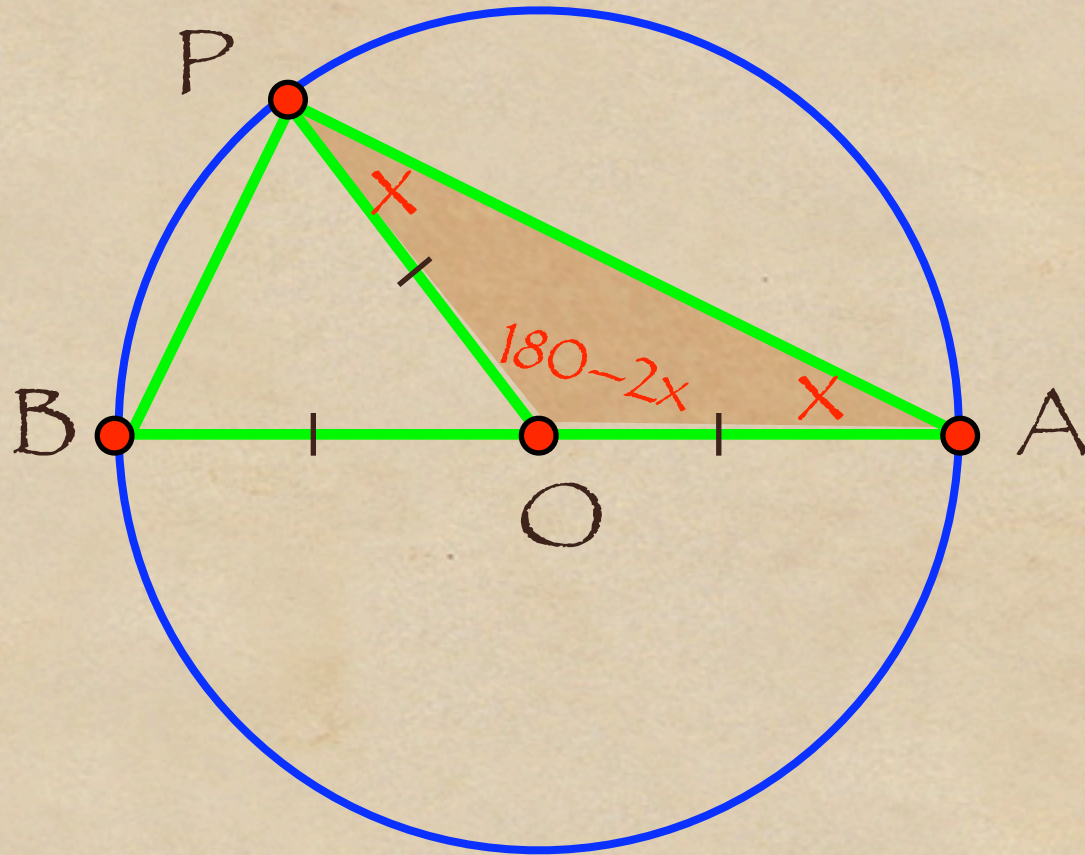
Triangles AOP and BOP are isosceles.

We do not know what angle OAP is so we let it be x .

Since the triangle is isosceles, the other base angle is also x , making the third angle

$180^\circ - 2x$.

We start by focusing on the one of the isosceles triangles.



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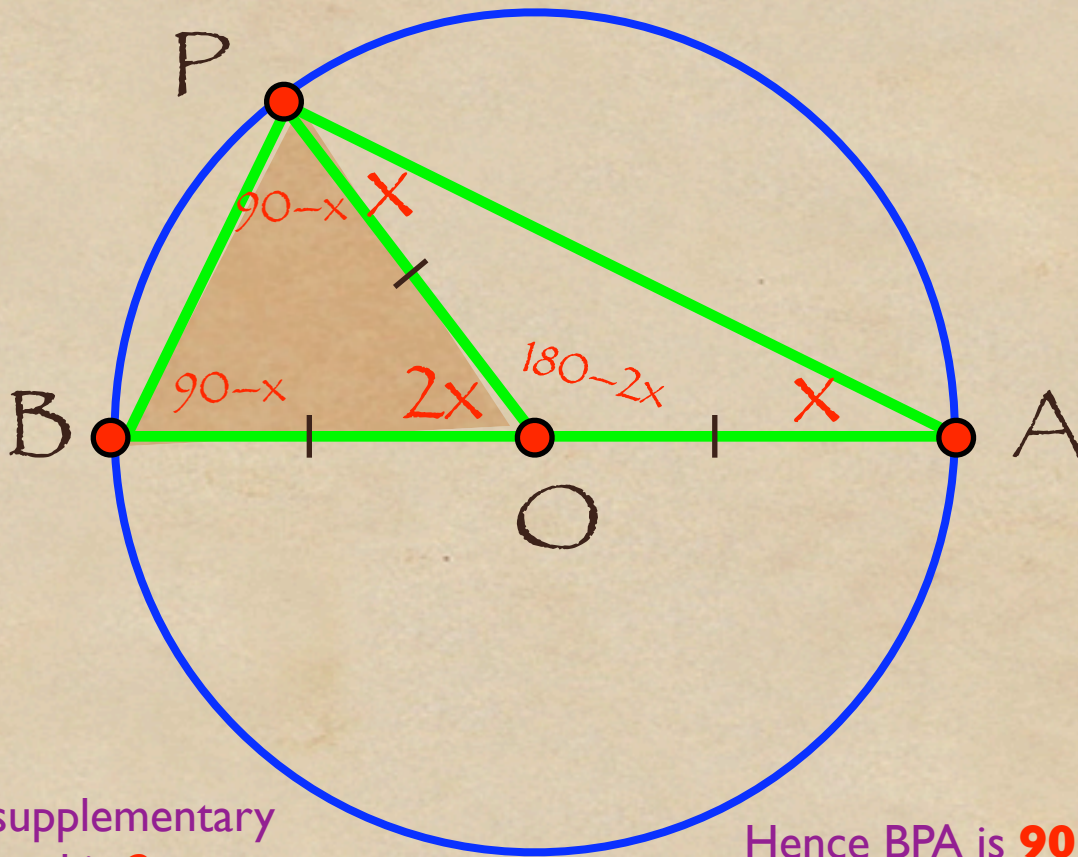
We do not know what angle OAP is so we let it be x .

Since the triangle is isosceles, the other base angle is also x , making the third angle $180^\circ - 2x$.

BOP is supplementary to AOP and is $2x$, making the two base angles $90^\circ - x$.

Hence BPA is $90^\circ - x$ plus x and is 90° , a right angle. QED

Now we focus on the other isosceles triangle.



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Circle O with a diameter AB and point P.

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OA, OP, and OB are radii of the circle and so are equal.

Triangles AOP and BOP are isosceles.

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